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Event Horizon as Alternative to Cosmological Singularity

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The general relativistic equations for a homogeneous isotropic metric of the most general form are considered. It is concluded that an event horizon is a possible alternative to a cosmological singularity for "usual" matter. This conclusion is illustrated by the case of a flat spatial universe.

It is known (Weinberg, 1972) that the homogeneous isotropic metric of most general form can be written in the following way:

$$ds^2 = -b(t)\,dt^2 + a^2(t)[d\mathbf{u}^2 + k(\mathbf{u}d\mathbf{u})^2/(1 - k\mathbf{u}^2)] \tag{1}$$

For spherical coordinates the expression (1) reduces to the form

$$ds^2 = -b(t)\,dt^2 + a^2(t)\{[R'^2/(1 - kR^2)]\,dr^2 + R^2(d\theta^2 + \sin^2\theta\,d\phi^2)\} \tag{2}$$

where $R^2(r) = \mathbf{u}^2$ (the prime denotes differentiation with respect to r).

With the exception of the degenerate case $R'(r) \equiv 0$, for the metric (2) the general relativistic equations $G^\mu_\nu = \kappa T^\mu_\nu$ have the form

$$(k + \dot{a}^2/b)/a^2 = \kappa\epsilon/3 \tag{3a}$$

$$\ddot{a}/a - \dot{a}\dot{b}/(2ab) = -\kappa b(\epsilon + 3P)/6 \tag{3b}$$

where $\epsilon = T^0_0$ and $-P = T^1_1 = T^2_2 = T^3_3$ (the dot denotes differentiation with respect to t).

It is assumed as a rule that the interval (2) can be reduced to an expression including $g_{00}(\bar{t}) \equiv -1$ with the help of the time coordinate transformation

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$\bar{t} = \int \sqrt{b}\, dt$. In this case from equation (3b) for $b \equiv 0$ and $b \equiv 1$ we conclude the inevitability of an initial singularity, insofar as the curve $a(t)$ is convex up [see the discussion in Weinberg (1972)].

One possible exception to the initial singularity is a transition to the de Sitter vacuum solution $\epsilon = -P \neq 0$ (Tolman, 1969). This solution is the "ideological foundation" for the inflationary cosmology (Starobinski, 1979; Guth, 1981).

The other possibility to avoid a cosmological singularity (for "usual" matter) is the presence of an event horizon $b_0 = 0$. In this case the above-mentioned transition to the coordinate $\bar{t}$ is inadmissible at $b = 0$.

Let us consider therefore the general case with arbitrary function $b(t)$.

According to equation (3b) the following asymptotic constraint exists near the event horizon between functions a and b: $\dot{a}^2 \sim b$. Hence, the event horizon corresponds to a minimum $a(t)$ and it is the initial point of the evolution of the universe.

In this case the matter of the universe can be "usual," i.e., obey the conditions

$$\epsilon \geq P \geq 0 \tag{4a}$$

$$0 \leq C_s^2 \equiv dP/d\epsilon \leq 1 \tag{4b}$$

Let us consider, for example, the flat spatial universe at

$$a = a_0 + \alpha t^2 \tag{5a}$$

$$b = \gamma t^2(1 + \beta t^2) \tag{5b}$$

where a_0, α, β, γ are positive constants.

Obviously, the initial point of the expansion of the universe $t = 0$ corresponds to the cosmological event horizon.

Substituting the expressions (5) into the system (3), we obtain

$$\kappa\epsilon = (12\alpha^2/\gamma)a^{-2}(1 + \beta t^2)^{-1} \tag{6a}$$

$$\kappa P = (4\alpha/\gamma)(\beta a_0 - \alpha)a^{-2}(1 + \beta t^2)^{-2} \tag{6b}$$

The conditions (4) for the functions (6) are fulfilled for the following correlation between the parameters:

$$1 \leq \beta a_0/\alpha \leq [3 + (73)^{1/2}]/4 \tag{7}$$

Thus, the event horizon is inevitable for "usual" matter as the alternative to a cosmological singularity.

Hence, the initial state of the universe can be considered as an untraversable wormhole (Morris *et al.*, 1988). This interpretation admits, for example, the exotic case of a topological transition from (3 + 1)-space to 4-space that agrees with the result of Ellis *et al.* (1992). Then it is possible that the "big bang" is the transformation of some space coordinate into a time coordinate. The analogous situation take place for the transition between Lorentzian and Euclidean wormholes (see, for example, Khalatnikov and Schiller, 1993).

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